INELASTIC BUCKLING OF FLAT PLATES

BY

ROMILOS TEODOSIADIS

B.S., Robert College, Istanbul, Turkey, 1945
M.S., University of Illinois, 1947

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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Introduction

Bijlaard is the first writer who made use of biaxial stress-strain relations to study inelastic buckling of flat plates. He separated the strain tensor into an elastic part and an inelastic part. The stresses corresponding to the elastic strains were expressed by equations of elasticity theory. Superimposed upon these stresses were inelastic stresses determined by equations of plasticity of an incompressible material.

Ilyushin developed a theory of buckling of flat plates that was based upon the Hencky-von Mises equations of plasticity for an incompressible material. This theory takes account of the anomalous behavior of the material during unloading. It is consequently a generalization of the double-modulus theory of columns. However, Ilyushin's theory leads to non-linear partial differential equations that impede the solutions of special problems.

Stowell simplifies Ilyushin's theory by employing the hypothesis that the stresses are uniquely determined by the strains. This viewpoint is also employed in the tangent-modulus theory of columns. Stowell accounted for the effect of Poisson's ratio by means of an empirical assumption.

In this thesis, Stowell's theory is generalized by means of Hencky's third assumption — the proportionality between volumetric strain and pressure. Accordingly, the effect of Poisson's ratio is derived.

Under the sponsorship of the Department of Theoretical and Applied Mechanics of the University of Illinois, a number of tests on the inelastic buckling of 24S-T aluminum-alloy plates have been conducted. Comparisons among the test results and the several theories are presented.

Development Of The Condition Of Buckling

Hencky postulated that the following conditions, which are true for an isotropic elastic body, remain valid for the strain hardening range:

a) The principal shearing stresses are proportional to the corresponding principal shearing strains — the factor of proportionality depending upon the state of strain.

b) The directions of the principal normal strains and the principal normal stresses coincide.

c) The volumetric strain is proportional to the pressure — the factor of proportionality being the same as in the elastic range.

With these assumptions, and the further assumption that the octahedral shearing stress is uniquely determined by the octahedral shearing strain, the following differential equation for buckling of plates has been derived:

$$C_1 w_{xxxx} - C_2 w_{xxxy} + 2C_3 w_{xxyy} - C_4 w_{xyyy} + C_5 w_{yyyy} - t/D\ (\sigma_x w_{xx} + \sigma_y w_{yy} + 2\tau_{xy} w_{xy}) = 0,$$

in which t is the thickness of the plate, and C_1, C_2, C_3, C_4, C_5 and D are constants that depend upon the applied stresses (σ_x, σ_y, τ_{xy}), Poisson's ratio, and properties of the compression-stress-strain curve of the material.

Buckling stresses are eigenvalues of the above equation. Solutions may be obtained for cases of uniaxial and biaxial compression by choosing values for σ_x and σ_y and solving for t. Various boundary conditions may be treated in this way. If shearing stresses are present, the eigenvalue problem is difficult, and the Rayleigh-Ritz method is preferable. Since the differential equation has been derived from a variational problem, there is no difficulty in applying the Rayleigh-Ritz method.

Experimental Work

A number of long, 24S-T aluminum-alloy plates with simply-supported edges were tested in uniaxial compression. Comparisons among the several theories and the test results are presented by means of curves whose abscissas are the width-to-thickness ratios, and whose ordinates are the buckling stresses. The agreement between theory and experiment is fair.

VITA

Romilos Teodosiadis was born in Istanbul, Turkey on February 6, 1926. He is a citizen of Turkey. In June, 1945 he received the degree of Bachelor of Science in Civil Engineering from Robert College, Istanbul, Turkey. In February, 1947, he received the degree of Master of Science in Civil Engineering from the University of Illinois.

During the three academic years between October, 1946 and June, 1949, he worked as a half-time research graduate assistant for the Department of Theoretical and Applied Mechanics of the University of Illinois. During the two summers included in the same period, he worked for two consulting engineering firms in Chicago, Illinois.